W-1/27

EECS 16B Section 2B

Main Topic: RC Circuits

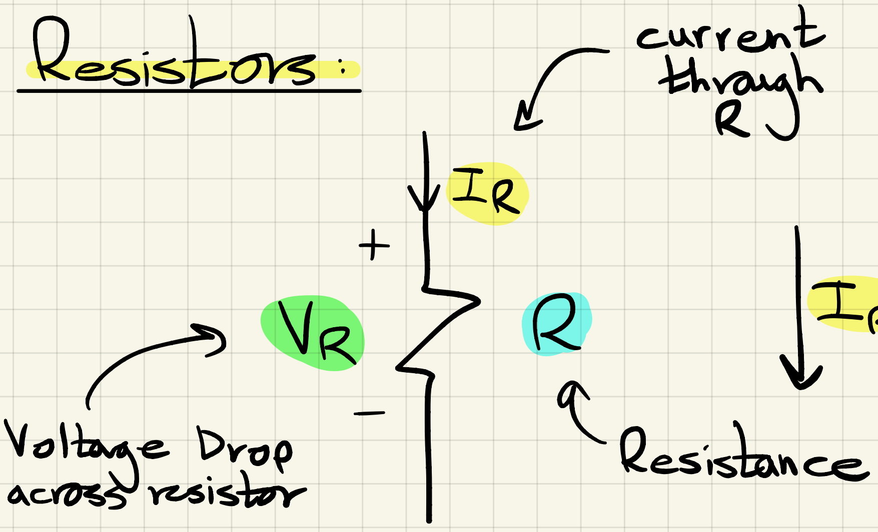
Administrivia:

- HW 2 due Fri, 1/29
- Anonymous Feedback:
  bit.ly/maxwell-16B-feedback-sp21

Agenda:

- Setup + Equations
- Q1a: Capacitors
- Q1b: KCL, KVL
- Q1c: Substitution
- Q1d: Solving Diff. Eq.
- Q1e: RC Circuit w/ voltage source

Resistors:



Ohm's Law:

$$V = IR$$

$$R = \frac{V}{I}$$

$$I = \frac{V}{R}$$

KCL:

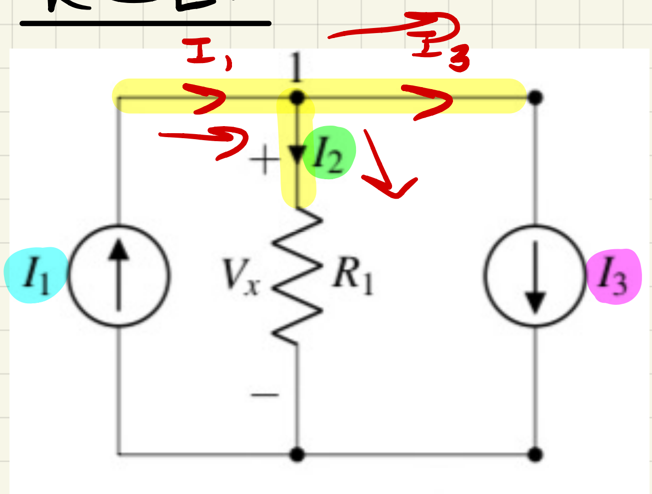


Figure 1: KCL Circuit

$$\Sigma I_{in} = 0 = \Sigma I_{out}$$

$$I_1 = I_2 + I_3$$

$$I_1 - I_2 - I_3 = 0$$

KVL:

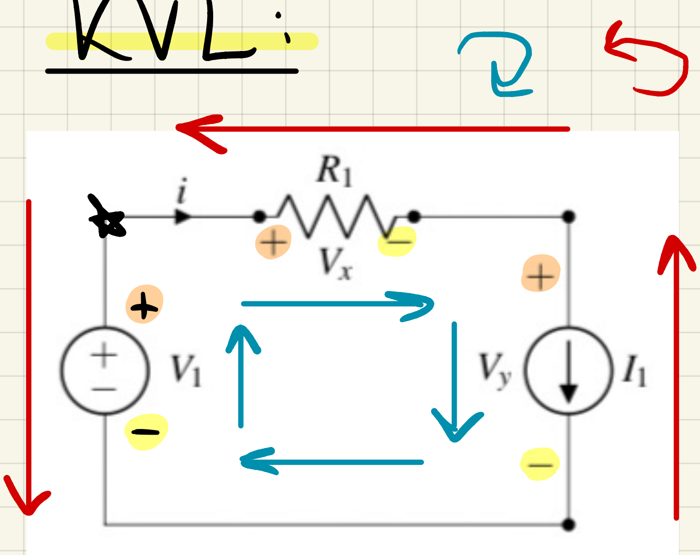


Figure 2: KVL Circuit

$$\Sigma V_{loop} = 0$$

+ ⟶ − : drop, negative

− ⟶ + : rise, positive

$$-V_1 + V_x + V_y = 0$$

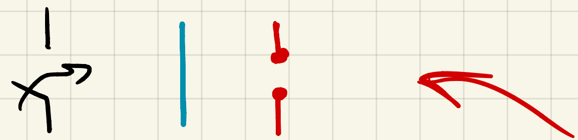
$$\parallel$$

$$-V_x - V_y + V_1 = 0$$

"Big Picture" so far

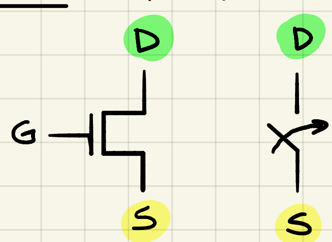
Logic gates are the building blocks of computing
→ Implemented w/ transistors

16A "transistor": a switch



16B: NMOS | PMOS

NMOS



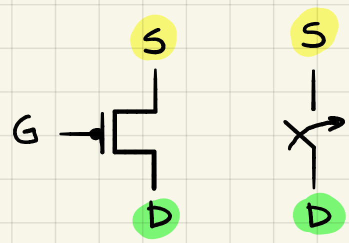
$V_{GS} \geq V_{tn}$ ($V_G - V_S$)

Threshold voltage for NMOS

NMOS = "Normal"

Input	Output
"High" = "1" = "Vdd"	"High" = "1" = "Vdd" = closed
"Low" = "0" = "Ground"	"Low" = "0" = "Ground" = open

PMOS



$V_{SG} \geq |V_{tp}|$ ($V_S - V_G$)

Threshold Voltage for PMOS

PMOS = "Peculiar"

Input	Output
"High" = "1" = "Vdd"	"Low" = "0" = "Ground" = open
"Low" = "0" = "Ground"	"High" = "1" = "Vdd" = closed

Key next step: Transistors are not perfect switches; we increase our model's complexity to be more realistic.

$P = I^2 R$

Resistor = Power Consumption

Capacitor = Time Delay

So, our transistor is no longer just wire; it has an "R" and "C", hence "RC Circuit".

Today is about analyzing these!

RC Circuits

In this problem, we will be using differential equations to find the voltage across a capacitor over time in an RC circuit. We set up our problem by first defining four functions over time: $I(t)$ is the current at time t, $V(t)$ is the voltage across the circuit at time t, $V_R(t)$ is the voltage across the resistor at time t, and $V_C(t)$ is the voltage across the capacitor at time t.

Recall from 16A that the voltage across a resistor is defined as $V_R = RI_R$ where I_R is the current across the resistor. Also, recall that the voltage across a capacitor is defined as $V_C = \frac{Q}{C}$ where Q is the charge across the capacitor.

(aka $Q = CV$)

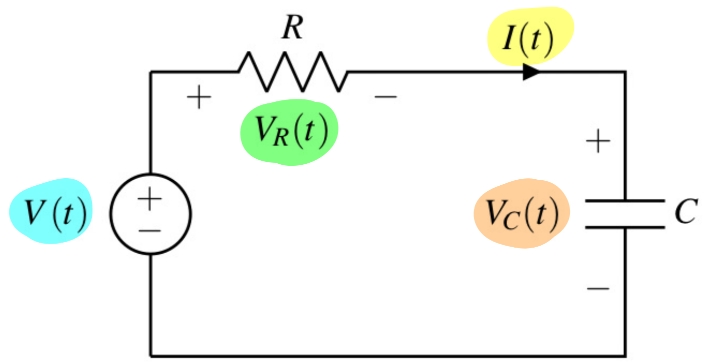


Figure 1: Example Circuit

(a) First, find an equation that relates the current across the capacitor $I(t)$ with the voltage across the capacitor $V_C(t)$.

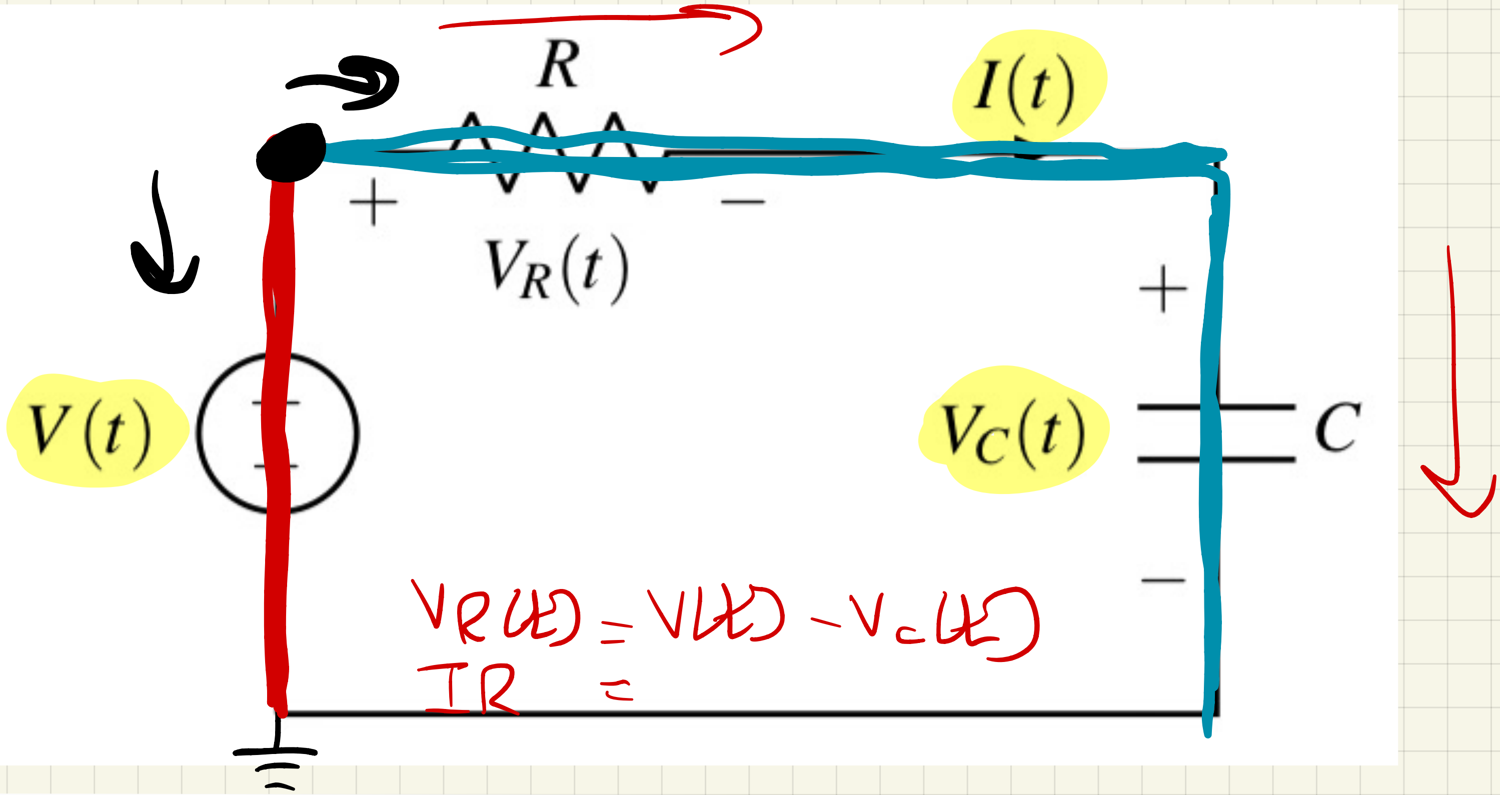
$$Q = CV$$

$$\frac{d}{dt}(Q = CV)$$

$$\left[\frac{d}{dt} Q = I \right]$$

$$I = C \frac{d}{dt} V$$

$$I_C(t) = C \frac{d}{dt} V_C(t)$$



(b) Write a system of equations that relates the functions $I(t)$, $V_C(t)$, and $V(t)$.

$$V(t) = V_R(t) + V_c(t)$$

$$\boxed{V_R = IR}$$

$$V(t) = I(t) \cdot R + V_c(t)$$

(c) So far, we have three unknown functions and only one equation, but we can remove $I(t)$ from the equation using what we found in part (a). Rewrite the previous equation in part (b) in the form of a differential equation.

$$I_c(t) = c \frac{d}{dt} V_c(t)$$

$$I_c(t) = I(t)$$

$$V(t) = I(t) \cdot R + V_c(t)$$

$$V(t) = c \frac{d}{dt} V_c(t) \cdot R + V_c(t)$$

$$\frac{d}{dt} V_c(t) = \frac{1}{Rc} (V(t) - V_c(t))$$

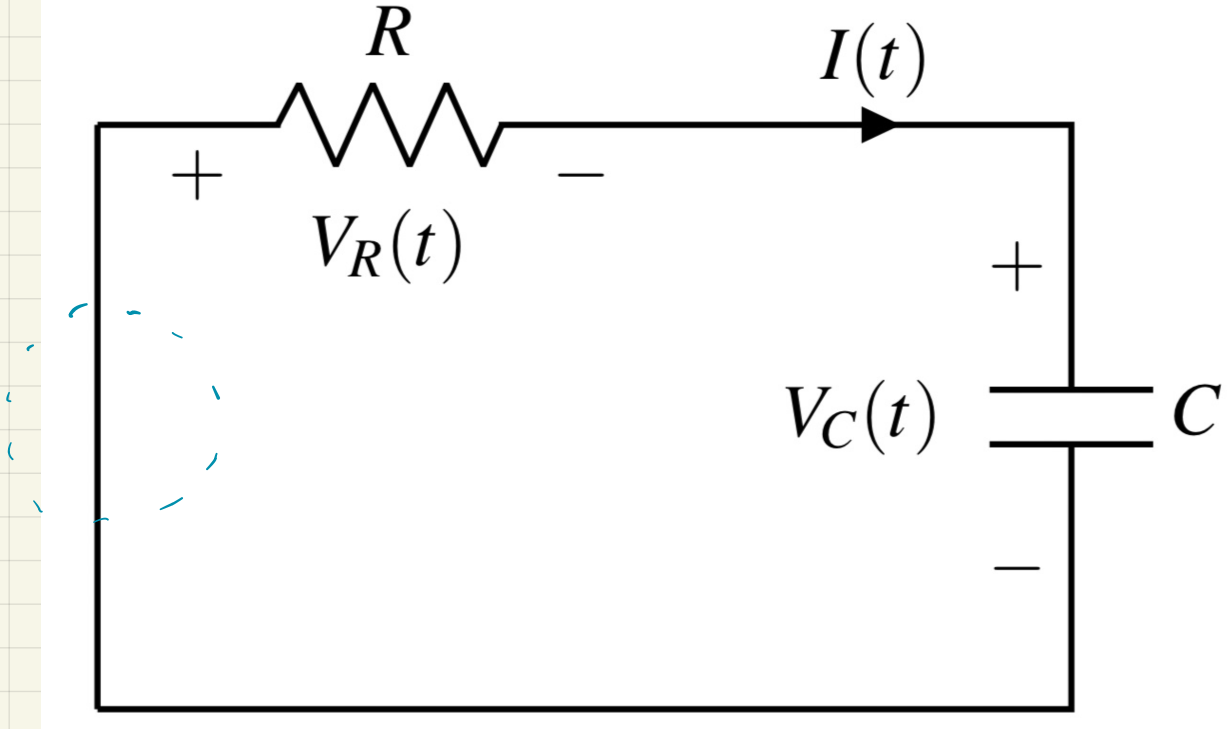


Figure 2: Circuit for part (d)

(d) Let's suppose that at $t = 0$, the capacitor is charged to a voltage V_{DD} ($V_C(0) = V_{DD}$). Let's also assume that $V(t) = 0$ for all $t \geq 0$. Solve the differential equation for $V_C(t)$ for $t \geq 0$.

$V_C(0) = V_{DD}$

$V(t) = 0$

$$RC\frac{d}{dt}V_C(t) = \cancel{V(t)}^{0} - V_C(t)$$

$$RC\frac{d}{dt}V_C(t) = -V_C(t)$$

$$\boxed{\frac{d}{dt}V_C(t) = -\frac{1}{RC}V_C(t)}$$

$Ae^{\lambda t}$

$$V_C(t) = Ae^{\lambda t}$$

$$\frac{d}{dt}V_C(t) = (Ae^{\lambda t})\lambda$$

$$= \lambda V_C(t)$$

$$V_C(t) = \underline{\underline{A}} e^{-\frac{1}{RC}t}$$

"Initial condition: $t=0$, $V_C(0)$"

$$V_C(0) = V_{DD}$$

$$V_{DD} = A e^{-\frac{1}{RC}(0)}$$

$$V_{DD} = A(1)$$

$$V_{DD} = A$$

$$\boxed{V_C(t) = V_{DD}\, e^{-\frac{1}{RC}t}}$$

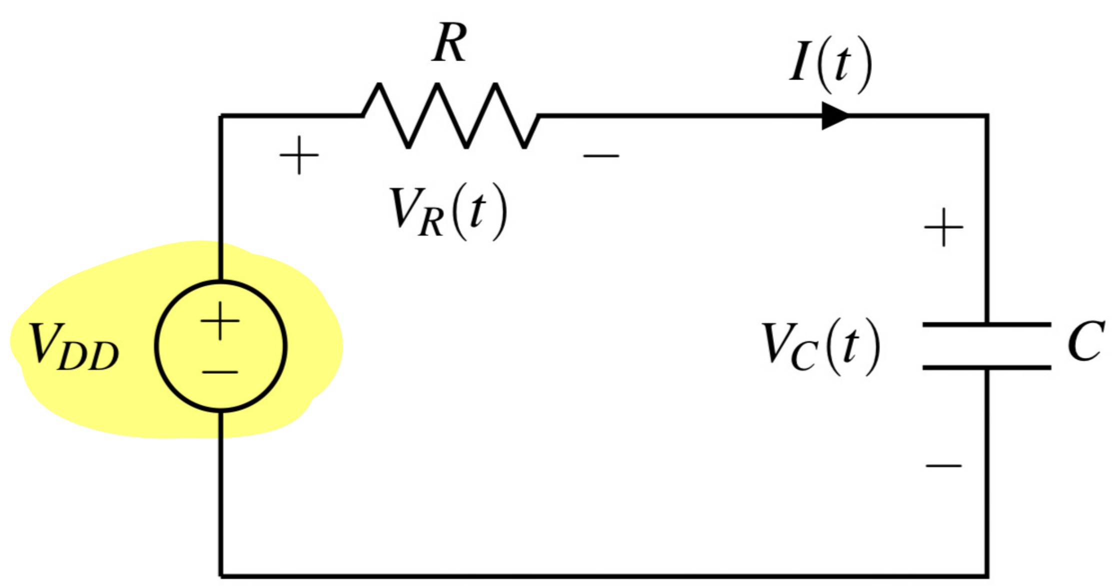


Figure 3: Circuit for part (e)

(e) Now, let's suppose that we start with an uncharged capacitor $V_C(0) = 0$. We apply some constant voltage $V(t) = V_{DD}$ across the circuit. Solve the differential equation for $V_C(t)$ for $t \geq 0$.

$V_c(0) = 0$

$V(t) = V_{DD}$

$$RC \frac{d}{dt} V_c(t) = V(t) - V_c(t)$$

$$RC \frac{d}{dt} V_c(t) = V_{DD} - V_c(t)$$

$$\frac{d}{dt} V_c(t) = \frac{V_{DD} - V_c(t)}{RC}$$

$$\left[\frac{d}{dt} V_c(t) = \lambda V_c(t) \right]$$

$$\tilde{V}_c(t) = V_c(t) - V_{DD}$$

$$\frac{d}{dt} V_c(t) = -\frac{1}{RC} \tilde{V}_c(t)$$

$$V_c(t) \neq \tilde{V}_c(t)$$

$$\frac{d}{dt}\tilde{V}_c(t) = \frac{d}{dt}V_c(t) - \cancelto{0}{\frac{d}{dt}V_{DD}}$$

$$\frac{d}{dt}\tilde{V}_c(t) = \frac{d}{dt}V_c(t)$$

$$\frac{d}{dt}\tilde{V}_c(t) = -\frac{1}{RC}\tilde{V}_c(t)$$

$$\tilde{V}_c(t) = Ae^{-\frac{1}{RC}t}$$

① Solve for A

② Go back to $V_c(t)$

$$V_c(t) - V_{DD} = Ae^{-\frac{1}{RC}t}$$

Initial condition $\rightarrow$ $$0 - V_{DD} = Ae^{0}$$

$$-V_{DD} = A$$

$$V_c(t) = V_{DD} - V_{DD}e^{-\frac{1}{RC}t}$$

$$V_c(t) = V_{DD}\left(1 - e^{-\frac{1}{RC}t}\right)$$

Recap:

(d)

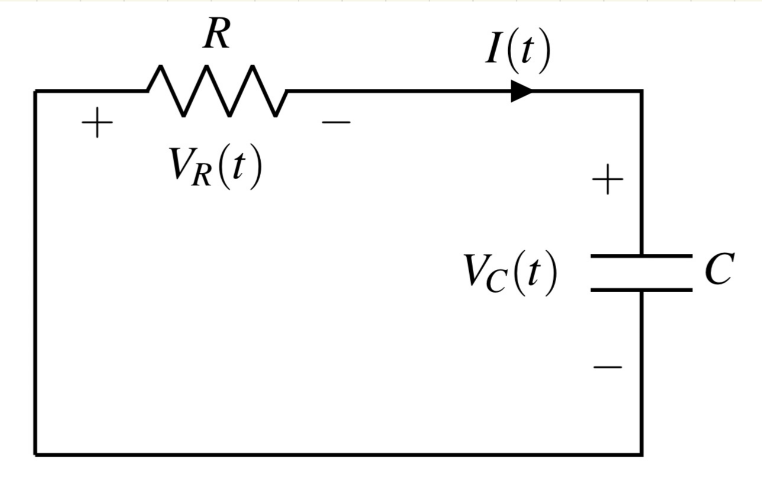


Figure 2: Circuit for part (d)

$$\frac{d}{dt} V_c(t) = -\frac{1}{RC} V_c(t)$$

$$\downarrow$$

$$V_c(t) = V_{DD}\, e^{-\frac{1}{RC}t}$$

"Homogenous Diff. eq."

(e)

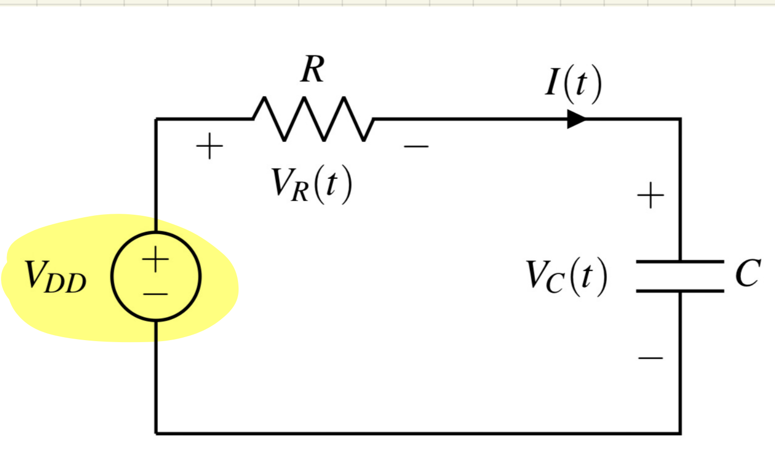


Figure 3: Circuit for part (e)

$$\frac{d}{dt} V_c(t) = -\frac{1}{RC} V_c(t) + \frac{V_{DD}}{RC}$$

$$\downarrow$$

$$V_c(t) = V_{DD}\left(1 - e^{-\frac{1}{RC}t}\right)$$

"Non-Homogenous Diff. eq."